Applied Statistics and Probability – Math 377
Fall 2020 Midterm 1

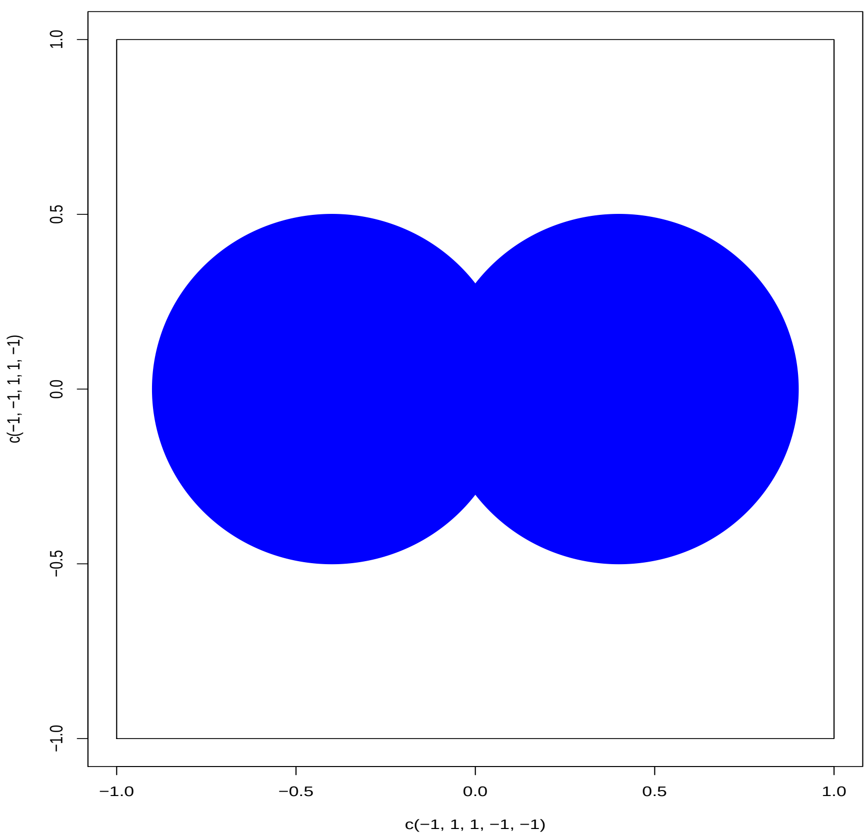
Answer all the following questions.

1. (10 points) Suppose X and Y are independent and they have Poisson distributions with mean 1.2 and 4.8 respectively. Write a R function that takes one input z and plots the conditional probability mass function of Y given $X + Y = z$, namely

$$f(k) := P(Y = k \mid X + Y = z)$$

   for all feasible values of k and returns the mean of the above conditional probability mass function. The x-axis should be labeled "values", y-axis should be labeled "conditional pmf", and the heading should be your name.
   (Hint: The distribution of $X + Y$ is also Poisson)

2. (4+3 points) Estimate the length of the boundary of the blue region in the above figure, which is the overlap of the discs having radius 0.5 and centers at (0.4,0) and (-0.4,0). Find an approximate 69% confidence interval for your estimate.



3. (5 points) A box contains w white balls and b black balls. A ball is chosen at random. The ball is then replaced, along with d more balls of the same color (as the chosen ball). Then another ball is drawn at random from the box. Find that probability that the second ball is white.

4. (5 points) Let X have a Chi-squared distribution with n degrees of freedom. Determine the smallest integer n such that the probability $P(X \geq 7)$ is at least 0.89.

5. (3+5 points) In a store, the time periods between successive arrivals of customers are independent and identically distributed (iid), and the common distribution is Exponential with mean 4 minutes. Suppose t is the time such that the probability that the 4th customer arrives after time t is 0.3. Obtain the value of t. Plot the probability density functions (over the interval $[0, 70]$) of Gamma(5,α) distributions for $\alpha = 4, 7, 10$. Use different colors and symbols and add a legend.

6. (5+5 points) Suppose X and Y are independent random variables with cumulative distribution functions

$$F_X(x) = \begin{cases} 1 - \exp(-4x^2) & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases} \quad F_Y(y) = \begin{cases} 1 - \exp(-3x^3) & \text{if } y \geq 0 \\ 0 & \text{if } y < 0 \end{cases}$$

respectively.
(a) Estimate the cumulative distribution function of $2X + Y$ at 3.
(b) Write a R function that takes one input z, which needs to be positive (an error message needs to be prompted if the input is negative), and returns an estimate of the probability density function of $2X + Y$ at z.